Analysis of the Symmetrical Modes for an Eccentrically Cladded Fiber

NICHOLAS M. METROU, JOHN D. KANELLOPOULOS, JOHN A. ROUMELIOTIS, AND JOHN G. FIKIORIS

Abstract —**This paper examines the core and the cladding modes of an eccentrically cladded three-layer dielectric waveguide. The solutions are specialized to small eccentricities, and exact closed-form expressions for the normalized deviations of the cutoff wavenumbers from those of the concentric case are determined. Numerical results for the symmetrical cladding modes of the fiber are given.**

I. Introduction

THE EVALUATION OF the cutoff wavenumbers of the symmetrical (Θ-independent) modes for an eccentrically cladded three-layer dielectric waveguide of circular cross section, is examined in this paper. A special analytical shape perturbation method, developed previously for waveguide and scattering eccentric problems [1],[2] is also used here for the analysis of both the cladding and core modes of the waveguide. The structure, illustrated in Fig. 1, is obviously a perturbation of the more commonly concentric, circular dielectric waveguide shown in Fig. 2 and treated in [3]. The method concludes with the following expression for the cutoff wavenumbers of the cladding modes:

$$K_{nm}(d)=K_{nm}(0)\left[1+g_{nm}(K_{nm}(0)d)^2\right] \quad (1)$$

in which the g_{nm}'s are given by exact closed-form expressions, whereas, for the core modes, the cutoff wavenumbers of the eccentric problem are shown to coincide, up to second order in kd included, with those of the concentric waveguide.

II. The Analysis

Referring to the waveguide of Fig. 1 and with assumed harmonic time dependence, we can expand the longitudinal field components $E_{z1}^1(P)$ and $H_{z1}^1(P)$ for region I in terms of cylindrical circular wave functions around the axis 0_1. A similar expansion is used for the $H_{z2}^1(P)$ and $E_{z2}^1(P)$ component in region II. Finally, the outside field E_{z3}^2, H_{z3}^2 in region III is expanded in terms of wave functions around the axis 0_2. The boundary conditions, to be satisfied for all

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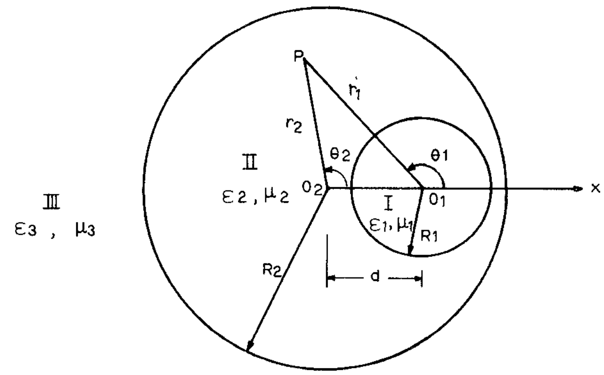

Fig. 1. Cross section of the eccentric circular waveguide.

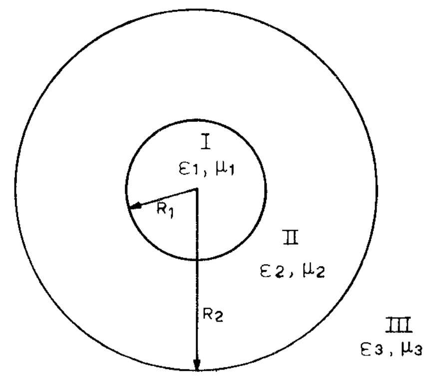

Fig. 2. Cross section of the concentric circular waveguide.

values of the azimuthal coordinate Θ, are

$$E_{z1}^1(P)=E_{z2}^1(P) \qquad E_{\Theta1}^1(P)=E_{\Theta2}^1(P)$$
$$H_{z1}^1(P)=H_{z2}^1(P) \qquad H_{\Theta1}^1(P)=H_{\Theta2}^1(P) \quad (2)$$

on the boundary surface (I)–(II) and

$$E_{z2}^2=E_{z3}^2 \qquad E_{\Theta2}^2=E_{\Theta3}^2$$
$$H_{z2}^2=H_{z3}^2 \qquad H_{\Theta2}^2=H_{\Theta3}^2 \quad (3)$$

on the boundary surface II–III, where the transverse components E_Θ^p, $H_\Theta^p(p=1,2)$ can be found in terms of E_z^p, H_z^p by well-known relations. In order to satisfy the boundary conditions (3), we reexpand the field components E_{z2}^1, H_{z2}^1 in terms of cylindrical circular wave functions around the

TABLE I
VALUES OF $u_{0m}(\epsilon_{r1}=2.341, \epsilon_{r2}=2.25)$

$q=R_1/R_2$	TM_{om}		TE_{om}	
	m=1	m=2	m=1	m=2
0.005	2.4048	5.5200	2.4048	5.5200
0.1	2.4030	5.5108	2.4048	5.5194
0.3	2.3898	5.4618	2.4030	5.4860
0.4	2.3798	5.4406	2.3996	5.4467
0.6	2.3577	5.4080	2.3834	5.3938
0.8	2.3376	5.3671	2.3548	5.3865
0.95	2.3253	5.3378	2.3298	5.3476

VALUES OF g_{om}

$q=R_1/R_2$	TM_{om}		TE_{om}	
	m=1	m=2	m=1	m=2
0.005	$4\cdot9\cdot10^{-6}$	$6\cdot4\cdot10^{-6}$	$-1\cdot10^{-6}$	$-1\cdot1\cdot10^{-6}$
0.1	$2\cdot3\cdot10^{-4}$	$4\cdot9\cdot10^{-4}$	$-3\cdot2\cdot10^{-4}$	$-6\cdot6\cdot10^{-4}$
0.3	$1\cdot7\cdot10^{-3}$	$1\cdot9\cdot10^{-3}$	$-2\cdot3\cdot10^{-3}$	$-5\cdot6\cdot10^{-4}$
0.4	$2\cdot7\cdot10^{-3}$	$1\cdot5\cdot10^{-3}$	$-3\cdot1\cdot10^{-3}$	$1\cdot9\cdot10^{-3}$
0.6	$4\cdot1\cdot10^{-3}$	$-1\cdot2\cdot10^{-3}$	$-2\cdot7\cdot10^{-3}$	$2\cdot8\cdot10^{-3}$
0.8	$4\cdot2\cdot10^{-3}$	$6\cdot2\cdot10^{-4}$	$1\cdot1\cdot10^{-3}$	$-2\cdot7\cdot10^{-3}$
0.95	$3\cdot2\cdot10^{-3}$	$7\cdot4\cdot10^{-4}$	$4\cdot7\cdot10^{-3}$	$-4\cdot0\cdot10^{-4}$

TABLE II
VALUES OF $u_{0m}(q=R_1/R_2=0.2)$

ϵr_1	ϵr_2	TM_{om}		TE_{om}	
		m=1	m=2	m=1	m=2
2.3	2.2999	2.4048	5.5200	2.4048	5.5200
2.3	2.29	2.4040	5.5116	2.4047	5.5191
2.4	2.2	2.3891	5.4474	2.4039	5.4998
2.4	2.1	2.3718	5.3669	2.4029	5.4745

VALUES OF $g_{om}(q = R_1/R_2 = 0.2)$

ϵr_1	ϵr_2	TM_{om}		TE_{om}	
		m=1	m=2	m=1	m=2
2.3	2.2999	$4\cdot7\cdot10^{-6}$	$5\cdot7\cdot10^{-6}$	$-1\cdot9\cdot10^{-7}$	$-1\cdot6\cdot10^{-6}$
2.3	2.29	$1\cdot1\cdot10^{-4}$	$6\cdot1\cdot10^{-4}$	$-1\cdot1\cdot10^{-4}$	$-1\cdot6\cdot10^{-4}$
2.4	2.2	$1\cdot9\cdot10^{-3}$	$3\cdot2\cdot10^{-3}$	$-2\cdot6\cdot10^{-3}$	$-3\cdot3\cdot10^{-3}$
2.5	2.1	$3\cdot8\cdot10^{-3}$	$6\cdot5\cdot10^{-3}$	$-5\cdot6\cdot10^{-3}$	$-6\cdot5\cdot10^{-3}$

axis 0_2 using the well-known translational addition theorems [4].

After straightforward steps similar to those in [1], one is able to obtain the following four sets of linear homogeneous equations:

$$\sum_{\nu=0}^{\infty} a_{p\nu} A'\nu + \sum_{\nu=1}^{\infty} \beta_{p\nu} B'\nu = 0 \qquad (p \geqslant 0)$$

$$\sum_{\nu=0}^{\infty} \gamma_{p\nu} A'_\nu + \sum_{\nu=1}^{\infty} \delta_{p\nu} B'_\nu = 0 \qquad (p > 0) \tag{4}$$

$$\sum_{\nu=0}^{\infty} \hat{a}_{p\nu} A_\nu + \sum_{\nu=1}^{\infty} \hat{\beta}_{p\nu} B_\nu = 0 \qquad (p \geqslant 0)$$

$$\sum_{\nu=0}^{\infty} \hat{\gamma}_{p\nu} A_\nu + \sum_{\nu=1}^{\infty} \hat{\delta}_{p\nu} B_\nu = 0 \qquad (p > 0) \tag{5}$$

where the A'_ν, B'_ν and A_ν, B_ν are the field expansion coefficients for the H^1_{z2} and E^1_{z2} components, respectively, and $a_{p\nu}$, $\beta_{p\nu}$, $\gamma_{p\nu}$, $\delta_{p\nu}$, $\hat{a}_{p\nu}$, $\hat{\beta}_{p\nu}$, $\hat{\gamma}_{p\nu}$, $\hat{\delta}_{p\nu}$ are complicated functions of the parameters of the problem. For nontrivial solutions, the two separate sets of equations (4) and (5) provide two characteristic equations in the form of infinite determinants from which the cutoff wavenumbers can be determined. These determinants are exactly the same in form as those in [1] for the cutoff wavenumbers of the Goubau waveguide. Consequently, the evaluation of the determinants may proceed along the lines suggested in [1], and one is able to obtain the development of the determinant up to order $(k_2 d)^2$, where k_2 is the wavenumber of the field in region II (cladding).

It is important to notice here, that in a cladded fiber there are two types of propagating modes, the cladding and core ones, corresponding to the appropriate cutoff condition [3]. In each case, this suggests a particular limiting

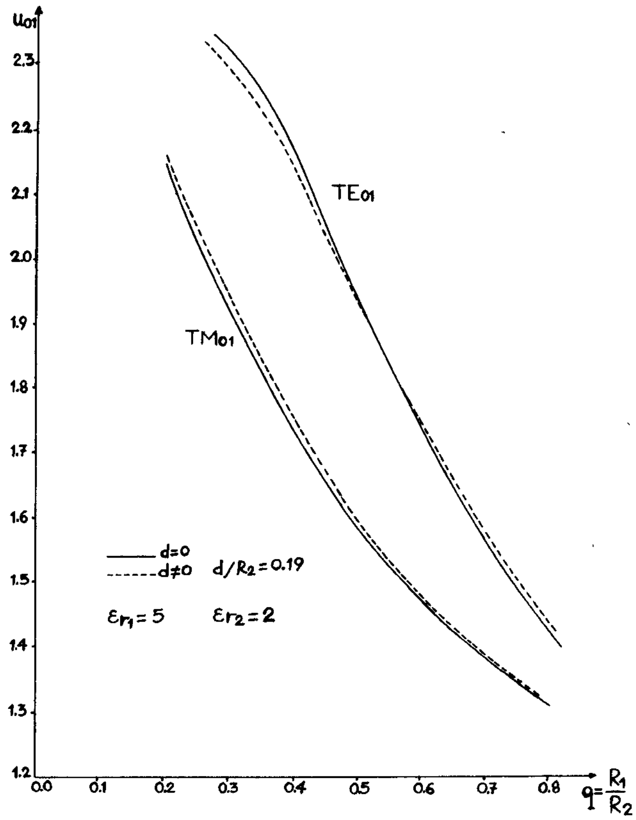

Fig. 3. Cutoff wavenumber variation $q=R_1/R_2$ for an eccentric cladded fiber ($\epsilon_{r1}=5$, $\epsilon_{r2}=2$, $d/R_2=0.19$).

procedure for the evaluation of the various terms in the determinant previously mentioned. It can further be shown that the cutoff wavenumbers $K_{nm}(d)$ of the cladding modes correspond one-to-one and have values very near the $K_{nm}(0)$ of the concentric case ($n \geqslant 0$, $m \geqslant 1$). The method concludes with the expression (1), in which the g_{nm}'s are given by exact closed-form expressions. The calculations are then focused on the symmetrical modes ($n=0$, TE and TM) which, with the exception of the HE_{11} mode, are the dominant ones in a dielectric waveguide [3]. Numerical results for various cases of such modes are given in the next section. On the other hand, the analysis for the core modes of a cladded fiber show that the cutoff wavenumbers of the symmetrical modes in the eccentric case are the same, at least up to the second order in *kd* included, as those of the concentric structure, a result easily explained by the fact that the cutoff condition concentrates the field of the propagating surface mode mainly inside and just outside the core and it is practically irrelevant what the geometry of the cross section is a little beyond the core, particularly for small *kd*.

III. Numerical Results

In the following tables we give the computed values of $u^0_{0m}=k_{0m}(0)\cdot R_2$ and the corresponding values of g_{0m} for both TM and TE modes and for several values of $q=R_1/R_2$ (Table I); also for various pairs of values $\epsilon_{r1}, \epsilon_{r2}$ (Table II). We see that for the chosen values of q (Table I) and pairs of dielectric constants ϵ_{r1}, ϵ_{r2} (Table II) the g_{01}'s for the next higher mode, TM_{01}, are positive. This indicates an increase in the operational bandwidth of the basic HE_{11} mode whose cutoff frequency can be shown to remain zero. The same remark has also been observed in the corresponding Goubau waveguide. Another useful observation is that the absolute values of g_{0m} for both the TM and TE modes become smaller as $q \to 0$ and $\epsilon_{r1}-\epsilon_{r2} \to 0$.

In Fig. 3, the dependence of u_{0m} versus $q=R_1/R_2$ for both concentric and eccentric cases is shown for a specific cladded fiber with $\epsilon_{r1}=5, \epsilon_{r2}=2$, and $d/R_2=0.19$, both for the TM_{01} and TE_{01} modes. The chosen value of d/R_2 satisfies the physical limitation $d/R_2 \leqslant 1-q$ for all values of q. The differences between eccentric and concentric cases appear small in these curves. We may remark, however, that the symmetrical Θ-independent modes will definitely be less affected by the eccentricity (which mainly disturb the Θ-dependence of the field) than the higher hybrid and Θ-dependent modes.

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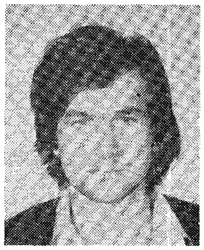

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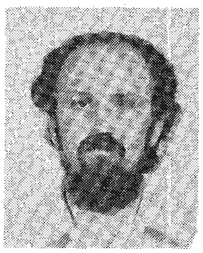

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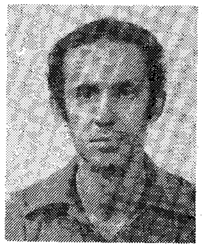

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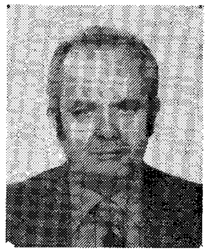

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Ridged Waveguides for Ultra-Broad-Band Light Modulators

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Abstract —**The electromagnetic field of the dominant mode propagating in the inhomogeneously dielectrically loaded double ridged waveguide is given in terms of a modal series expansion. The numerical evaluation of the propagation constant reveals a remarkably linear dispersion diagram in close agreement with measurements performed in the 8–40-GHz range. Based on this analysis, the bandwidth of a ridged waveguide CO_2-laser modulator is calculated to exceed 40 GHz, when a 25-mm long CdTe crystal is used as electrooptic material.**

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I. Introduction

ELECTROOPTICALLY mixing a fixed-frequency CO_2 laser with a frequency-tunable microwave signal yields continuously tunable laser sidebands in the infrared. In this way, the tunability of the microwave signal is transferred to the IR wavelength region from about 9–11 μm. Moreover, the accurate sideband frequency can be de-

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